

# Origin and Avoidance of Spurious Solutions in the Transverse Resonance Method

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**Abstract**—In the context of transverse resonance method, a criterion is established for the choice of trial functions introduced in Galerkin's method: this criterion allows to avoid the appearance of spurious solutions in the whole region of a propagation diagram and guarantees, at the same time, a good precision for the true solutions.

## I. INTRODUCTION

MOST OF the research dealing with the problem of spurious solutions has been developed in the context of finite-element method and many options for avoiding these nonphysical solutions have been presented in scientific literature [1]–[4]: the fundamental cause of spurious modes lies in the inaccurate approximation of the zero eigenvalue and the corresponding eigenfunctions [5]. The enforcement of the divergence-free constraint on the trial functions allows to suppress spurious solutions present in the initial formulation [2], [6].

Up to now, no scientific communication, at least to our knowledge, has dealt with the origin and the avoidance of spurious solutions in the transverse resonance method. This method, particularly well-adapted to the study of multilayered structures, is used very often to characterize dispersion phenomena in planar transmission lines [7]–[13]: the size of matrices resulting from the transverse resonance condition is reduced considerably compared with the usual finite-element method. Meanwhile, spurious solutions may be encountered in the numerical treatment of the transverse resonance method which are difficult to distinguish from the true propagation constants and which hinder the systematic investigation of physical solutions: the origin of these embarrassing solutions is very obscure and therefore their avoidance a priori seems to be quite difficult.

In this paper, the problem of spurious solutions is studied in the context of the transverse resonance method. We show that the characterization of dispersion phenomena in planar transmission lines gives rise to a resonance condition which has theoretically a solution for an infinite propagation constant. This nonphysical solution seems to be the origin of spurious solutions in the numerical resolution of dispersion problem: as a matter of fact, we show that the basic cause of spuri-

ous modes lies in the inaccurate approximation of the field belonging to this infinite- $\beta$  solution.

In order to describe well the fields belonging to the infinite- $\beta$  solution, a criterion for the choice of trial functions used in the Galerkin's method is rigorously found. In the E-field formulation, the tangential components of the electric field in the discontinuity plane are expanded over trial functions: the key step for the elimination of spurious solutions is the enforcement of zero curl of the E-field along the axis normal to the discontinuity plane. A similar criterion can be established in the H-field formulation, where we have to force the curl of the magnetic field to zero along the axis normal to the discontinuity plane.

The numerical applications of our general theoretical study is divided in two parts. First, in the case of unilateral finline, we consider the behavior of spurious and physical solutions with respect to the number of trial functions which do not satisfy the criterion deduced in the theoretical approach. These results illustrate the existence of an infinite solution for the propagation constant: the inaccurate description of the field belonging to this nonphysical solution generates spurious solutions.

In the second part of the numerical application, we show that a appropriate choice of trial functions satisfying the above mentioned criterion suppresses the spurious solutions and guarantees, at the same time, a good precision for the physical solutions.

## II. THEORETICAL APPROACH

In order to illustrate the theoretical developments, consider the general unilateral transmission line of Fig. 1. The metal on the interface is distributed arbitrarily.

The dielectric substrate of Fig. 1 is assumed to be isotropic and homogeneous. Losses in the dielectric and in the conductors as well as the metal thickness are neglected.

### A. Transverse Resonance Method

The analysis of planar structures using the transverse resonance method has been the subject of numerous publications [7]–[13]. In this study, the application of the method to the characterization of dispersion phenomena in the planar transmission line of Fig. 1 will be presented in a brief development. Note only that the operator formalism is used to reinforce the systematic character of this method: it is a concise and clear way to treat the well-known relationships between the electromagnetic fields, deduced from the equivalent transmission line of the structure.

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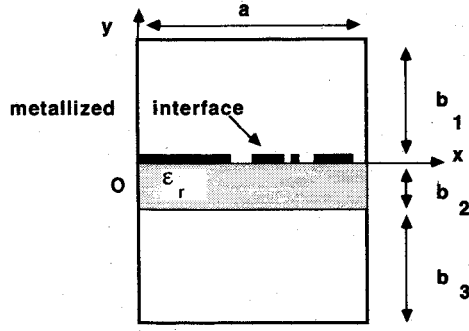


Fig. 1. Cross-sectional view of a unilateral planar transmission line.

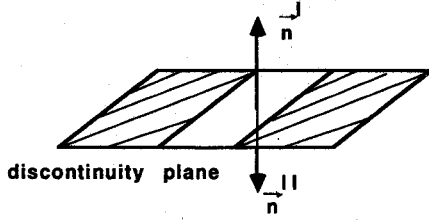


Fig. 2. Presentation of current density.

The first point of this method consists in the determination of the equivalent transmission line of the considered planar structure in the  $y$ -direction [13]. Let  $E$  represent the electric field in the metallized interface (called discontinuity plane, shown in Fig. 2) and define the current density  $J$  in this plane by the following relation:

$$\begin{aligned} \vec{J} &= \vec{H}^I \times \vec{n}^I + \vec{H}^{II} \times \vec{n}^{II} \\ \text{Thus } \vec{J} &= \vec{J}^I + \vec{J}^{II} \\ \text{with } \vec{J}^i &= \vec{H}^i \times \vec{n}^i \quad \forall i \in \{I, II\} \end{aligned} \quad (1)$$

Then, we can easily establish the equivalent transmission line of the studied structure shown in Fig. 3.

Note that the electric field  $E$  is the fundamental unknown: we call it the “adjustable source”. We have to solve the two continuity relation imposed to the electromagnetic fields in the discontinuity plane, that is:

$$\vec{E} = 0 \quad \text{on the metal} \quad (2)$$

$$\vec{J} = 0 \quad \text{elsewhere} \quad (3)$$

In the  $E$ -field formulation, these two equations are expressed only in terms of the adjustable source  $E$ :

$$\vec{E} = 0 \quad \text{on the metal} \quad (4)$$

$$\hat{Y}\vec{E} = 0 \quad \text{elsewhere} \quad (5)$$

where  $\hat{Y}$  is the total admittance operator viewed by the discontinuity plane (The H-field formulation is discussed later).

In order to satisfy (4), we just expand the electric field  $E$  on a basis  $g_p$  which element are zero on the metallic part of the discontinuity plane.

Next, we determine the matrix representation of the admittance operator  $Y$  on this basis (Galerkin's method). The general term of this matrix can be written under the following

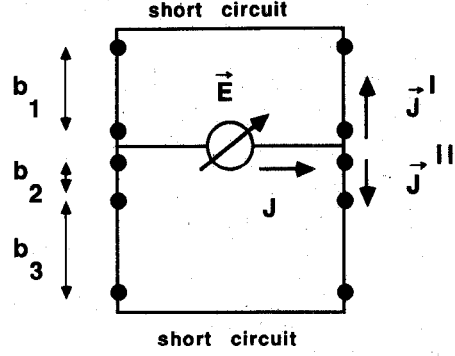


Fig. 3. Transmission line equivalent of planar structure shown Fig. 1.

form:

$$\begin{aligned} Y_{pq} &= \sum_n \langle \vec{g}_p, \vec{f}_n^{\text{TE}} \rangle Y_n^{\text{TE}} \langle \vec{f}_n^{\text{TE}}, \vec{g}_q \rangle \\ &\quad + \langle \vec{g}_p, \vec{f}_n^{\text{TM}} \rangle Y_n^{\text{TM}} \langle \vec{f}_n^{\text{TM}}, \vec{g}_q \rangle \\ Y_n^{\text{TE}} &= (Y_n^I)^{\text{TE}} + (Y_n^{II})^{\text{TE}} \end{aligned}$$

and

$$Y_n^{\text{TM}} = (Y_n^I)^{\text{TM}} + (Y_n^{II})^{\text{TM}} \quad \forall n \quad (6)$$

and the inner product:

$$\langle \vec{f}_m, \vec{f}_n \rangle = \int_0^a \vec{f}_m^* \cdot \vec{f}_n dx = \delta_{mn}$$

$\delta_{mn}$  is the delta Kronecker.

The analytical expressions of the well-known  $\text{TM}_y$  and  $\text{TE}_y$  mode admittances  $Y_n^I$  and  $Y_n^{II}$  are given below [12]:

$$\begin{aligned} Y_n^I &= Y_n(1) \coth(p_n(1)b_1) \\ Y_n^{II} &= Y_n(\epsilon_r) \frac{Y_n^o + Y_n(\epsilon_r) \tanh(p_n(\epsilon_r)b_2)}{Y_n(\epsilon_r) + Y_n^o \tanh(p_n(\epsilon_r)b_2)} \end{aligned} \quad (7)$$

with

$$Y_n^o = Y_n(1) \coth(p_n(1)b_3),$$

and mode admittance  $Y_n(\epsilon_r)$  :

$$\text{TE}_y \text{ mode : } \frac{p_n(\epsilon_r)}{j\omega\mu_o}; \quad \text{TM}_y \text{ mode : } \frac{j\omega\epsilon_o\epsilon_r}{p_n(\epsilon_r)} \quad (8)$$

with

$$p_n^2(\epsilon_r) = \left(n \frac{\Pi}{a}\right)^2 + \beta^2 - k_o^2 \epsilon_r$$

and

$$k_o^2 = \omega^2 \mu_o \epsilon_o.$$

The determinant of  $(\hat{Y})$  is put equal to zero in order to ensure the existence of non-trivial solutions for (5): this is in fact, a resonance condition which allows to calculate the unknown propagation constant from a variational form.

### B. Existence of an Infinite Solution for the Propagation Constant $\beta$

By applying the transverse resonance method, we demonstrate the existence of an infinite solution for the dispersion problem. In other words, we demonstrate the fact that:

$$\begin{aligned} (\hat{Y})(\vec{E}) &\rightarrow 0 \\ \beta &\rightarrow \infty \end{aligned}$$

The symbol  $(Y)$  defines the matrix representation of the admittance operator on the basis  $g_p$  and  $\beta$  denotes the unknown propagation constant along the  $z$ -axis. We can write:

$$(\vec{E}) = \sum_p x_p \vec{g}_p$$

and the boundary condition (5):

$$(\hat{Y})(\vec{E}) = 0 \Rightarrow \sum_q Y_{pq} x_q = 0 \quad \forall p \quad (9)$$

with  $Y_{pq}$  given above by (6). Since

$$\begin{aligned} Y_n^{\text{TE}} &\rightarrow \infty \\ \beta &\rightarrow \infty \end{aligned}$$

and

$$\begin{aligned} Y_n^{\text{TM}} &\rightarrow 0 \\ \beta &\rightarrow \infty \quad \forall n \end{aligned} \quad (10)$$

we can find a solution  $E_\infty$  for (9) when  $\beta$  is infinite: it is sufficient to take:

$$(\vec{E}_\infty) = \sum_p x_p^\infty \vec{g}_p$$

with

$$\langle \vec{g}_p, \vec{f}_n^{\text{TE}} \rangle = 0 \quad \forall n, p$$

that is to say  $E_\infty$  expanded over a  $\text{TM}_y$  basis satisfies the boundary conditions for infinite  $\beta$ .

Thus, we obtain the following result: infinite- $\beta$  is a possible theoretical solution of the dispersion problem. This nonphysical solution satisfies the fundamental boundary conditions (5) and is transverse magnetic along  $y$ -axis.

In other words, from the Maxwell equation, we can write:

$$(\vec{\nabla} \times \vec{E}) \cdot \vec{y} = -j\omega\mu\vec{H} \cdot \vec{y} = 0$$

since

$$H_y = 0$$

which leads to

$$\begin{aligned} \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= 0 \\ \Rightarrow -j\beta E_x &= \frac{\partial E_z}{\partial x} \end{aligned}$$

Therefore, in order to fully described all the solution of the boundary problem, that is to say the electric field belonging to the physical propagation constants and the  $E_\infty$  field associated

to the infinite- $\beta$  solution, we have to choose trial functions  $\phi_x^m(x)$  and  $\phi_z^m(x)$  so that:

$$\phi_x^m(x) \propto \frac{\partial \phi_z^m}{\partial x}(x) \quad \forall m \quad (11)$$

with

$$\vec{E} \left( \sum_m e_x^m \phi_x^m(x) \right) \left( \sum_m e_z^m \phi_z^m(x) \right) \quad (12)$$

where  $e_x^m$  and  $e_z^m$  are the components over trial functions  $\phi_x^m$  and  $\phi_z^m$  respectively.

*Comments:* 1) In the H-field version of the method, the current density is taken as the fundamental unknown of the problem.

From (2) and (3), the new equations to be solved will be the followings:

$$\hat{Z}\vec{J} = 0 \quad \text{on the metal} \quad (13)$$

$$\vec{J} = 0 \quad \text{elsewhere} \quad (14)$$

instead of (4) and (5) considered in the E-field version, in which  $Z$  is the total impedance operator viewed by the discontinuity plane. Thus, we have to solve the system equations:

$$(\hat{Z})(\vec{J}) = 0 \Rightarrow \sum_q Z_{pq} x_q = 0 \quad \forall p$$

with

$$(\vec{J}) = \sum_p x_p \vec{g}_p$$

and

$$\begin{aligned} Z_{pq} &= \sum_n \langle \vec{g}_p, \vec{f}_n^{\text{TE}} \rangle \frac{1}{Y_n^{\text{TE}}} \langle \vec{f}_n^{\text{TE}}, \vec{g}_q \rangle \\ &\quad + \langle \vec{g}_p, \vec{f}_n^{\text{TM}} \rangle \frac{1}{Y_n^{\text{TM}}} \langle \vec{f}_n^{\text{TM}}, \vec{g}_q \rangle \end{aligned}$$

Therefore, we obtain a similar criterion of (11) for the choice of trial functions  $\theta_x^m$  and  $\theta_z^m$  expanding the current density in the discontinuity plane: we could easily demonstrate that a  $\text{TE}_y$  condition must be satisfied in order to fully describe all the mathematical solutions of (13) and (14). This condition can be expressed in terms of the current density. In fact, we have:

$$(\vec{\nabla} \times \vec{H}) \cdot \vec{y} = j\omega\epsilon\vec{E} \cdot \vec{y} = 0$$

since

$$E_y = 0$$

with

$$\vec{J} = \vec{H} \times \vec{y}$$

Therefore:

$$\begin{aligned} \frac{\partial J_x}{\partial x} - \frac{\partial J_z}{\partial z} &= 0 \\ \Rightarrow -j\beta J_z &= \frac{\partial J_x}{\partial x} \end{aligned}$$

so the current density in the discontinuity plane must be expanded over trial functions  $\theta_x^m$  and  $\theta_z^m$  which do satisfy the following relationship:

$$\theta_z^m(x) \propto \frac{\partial \theta_x^m}{\partial x}(x) \quad \forall m \quad (15)$$

with

$$\vec{J} \left( \begin{array}{c} \sum_m i_x^m \theta_x^m(x) \\ \sum_m i_z^m \theta_z^m(x) \end{array} \right) \quad (16)$$

where  $i_x^m$  and  $i_z^m$  are the components over trial functions  $\theta_x^m$  and  $\theta_z^m$  respectively.

Note that this result has been used for other reasons by Jansen in [18] for the characterization of dispersion phenomena in single and coupled microstrip lines.

2) An analogy can be made between our approach and the one used in the finite-element method: the basic cause of spurious modes in the latter method lies in the inaccurate approximation of the zero eigenvalues and the corresponding eigenfunctions [5]. These spurious modes do not satisfy the free divergence Maxwell equation. Indeed, the equation to be solved is:

$$(\vec{\nabla} \times \vec{\nabla} \times \vec{E}) - k_o^2 \vec{E} = 0 \quad (17)$$

In the case of isotropic and homogeneous media, by taking the divergence of (17) we obtain:

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\nabla} \times \vec{E}) - k_o^2 \vec{\nabla} \cdot \vec{E} = 0$$

which gives

$$k_o^2 \vec{\nabla} \cdot \vec{E} = 0 \quad (18)$$

So, (18) yields mathematically a non zero divergence of the electric field when  $k_o$  is equal to zero. The divergence of the eigenfunctions belonging to the zero eigenvalue is not zero.

The key step for the elimination of spurious modes is the enforcement of the zero divergence of the vector fields used for the fields description [2]–[6]. This static solution for  $\omega = 0$  is obviously a uninteresting one, but must be mathematically well-described, otherwise it takes a finite value and may appear as spurious solution in the investigation domain of physical solutions.

In the context of transverse resonance method, we formulate an analogous principle: since infinite- $\beta$  is a solution of the boundary equations to be solved, we have to describe simultaneously the  $E_\infty$  field corresponding to this solution and the  $E$  field belonging to the physical solutions. In other words, we have to choose correct trial functions which have to be appropriate for taking into account the characteristics of the true solutions and the requirement for infinite- $\beta$  solution. If this choice is not made, that is if the trial functions do not allow to describe the theoretical  $E_\infty$  field, the infinite- $\beta$  solution will take a very embarrassing finite value in the numerical treatment (becoming visible as a zero determinant in the domain of real solutions) and will hinder the investigation of physical solutions.

In this manner, in order to avoid the spurious solutions—or at least, to numerically remove these solutions far away from the physical ones—it seems judicious to choose trial functions satisfying the criterion (11) in the E-field formulation or criterion (15) in the H-field formulation.

Note that the application of the criterion (11) and (15) does not depend on the structure configuration. As matter of fact, the criterion (11) (resp. (15)) is deduced from the zero  $\text{TM}_y$  mode admittance (resp. zero  $\text{TE}_y$  mode impedance) when  $\beta$  is infinite: this property is quite general and, therefore, the criterion is applicable for all kinds of planar transmission lines.

### III. NUMERICAL RESULTS AND DISCUSSION

#### A. Numerical Properties of Detected Spurious Solutions

Let  $N_x$  and  $N_z$  represent the number of trial functions along the  $x$  and  $z$ -axis respectively, being inevitably finite numbers for the numerical requirements. The values of these parameters are conditioned by the convergence criterion on the solution for the propagation constant  $\beta$ .

The analytical expression of trial functions is given in such a way to ensure a good accuracy of obtained results, to build a well-conditioned matrix and to calculate easily the matrix elements [14].

Usually, in order to reduce the matrix size, the edge effects, that is to say the tendency of the normal electric field component to a metallic edge to become infinite near this edge, are taken into account by the choice of appropriate trial functions [15].

Take the example of a unilateral finline: the metallization thickness is assumed to be zero and allows us to choose the following usual trial functions for the electric field in the slot:

$$\begin{aligned} \phi_x^m(x) &= \frac{\cos(m-1)\frac{2\pi}{w}\left(x - \frac{a}{2}\right)}{\sqrt{\left(\frac{w}{4}\right)^2 - \left(x - \frac{a}{2}\right)^2}} \\ &\quad \forall m \in \{1, 2, 3, \dots, N_x\} \\ \phi_z^k(x) &= \sin(k-1)\frac{2\pi}{w}\left(x - \frac{a}{2}\right) \\ &\quad \forall k \in \{1, 2, 3, \dots, N_z\} \end{aligned} \quad (19)$$

where  $w$  designates the slot width. Thus, one or two trial functions are enough to obtain good numerical results for the propagation constant [16].

But, with these acceptable solutions, we detect another solution, called  $\beta_\infty$ , which has a surprising behavior versus the number of trial functions.

As a matter of fact, for a given number of modes, there exists a number  $N = N_x = N_z$  of trial functions beyond which this solution takes increasingly larger values (Fig. 4): The greater is the number  $N$  of trial functions, the better are described all the solutions of the boundary problem, especially the infinite- $\beta$  solution. Thus numerical solution  $\beta_\infty$  is removed to infinity for a sufficiently large number of trial functions.

The transverse Magnetic nature along the  $y$ -axis of the infinite- $\beta$  solution can be illustrated by the calculation of the

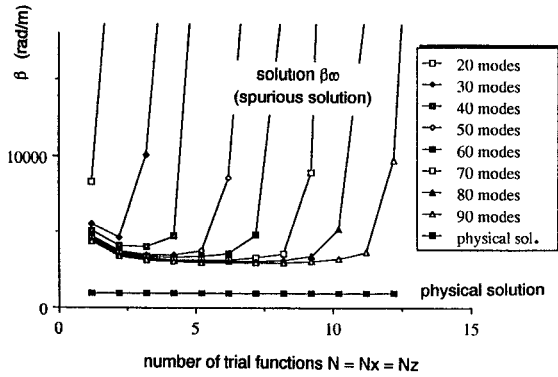


Fig. 4. The two numerical solutions of the resonance condition versus the number of trial functions in the case of a unilateral finline:  $a = 3.556$  mm,  $b_1 = 3.556$  mm,  $b_2 = 0.254$  mm,  $b_3 = 3.302$  mm,  $\epsilon_r = 2.22$  with a frequency of 30 GHz.

mode magnitudes of spurious solutions  $\beta_\infty$ : take a very large value for solution of the resonance condition  $\det(Y) = 0$  and determine the true solution of this resonance condition. As an example, in the case of a unilateral finline, with 70 modes and 11 trial functions, we give the mode magnitude spectrum for these two kinds of solutions (Fig. 5). It can be noted that the solution is principally Transverse Magnetic, since the  $TE_y$  mode magnitudes are negligible compared with  $TM_y$  mode magnitudes.

### B. Avoidance of Spurious Solutions

Applying the criterion (11) with the trial functions along  $x$ -axis mentioned above:

$$\phi_x^m(x) = \frac{\cos(m-1)\frac{2\pi}{w}\left(x - \frac{a}{2}\right)}{\sqrt{\left(\frac{w}{4}\right)^2 - \left(x - \frac{a}{2}\right)^2}} \quad \forall m \in \{1, 2, 3, \dots, N_x\}$$

and

$$\phi_z^m(x) = \int_0^x (\phi_x^m(x) - A_m) dx \quad (20)$$

yields trial functions along  $z$ -axis which have not to be expressed analytically. Actually, the matrix representation of the admittance operator needs only the determination of the inner products  $\langle \phi_x^m, f_{nx} \rangle$ , since the other inner products  $\langle \phi_z^m, f_{nz} \rangle$  can be easily deduced by integration by part:

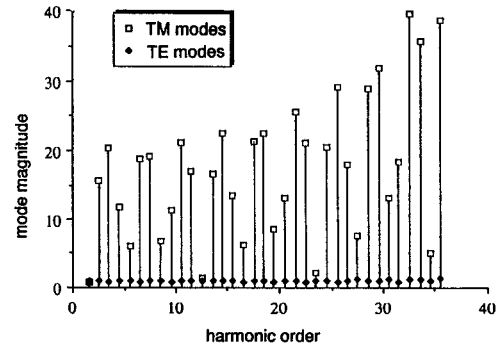
$$\begin{aligned} \langle \phi_x^k, f_{nx} \rangle &= \langle \partial_x \phi_z^k, f_{nx} \rangle \\ &= -\langle \phi_z^k, \partial_x f_{nx} \rangle = \alpha \langle \phi_z^k, f_{nz} \rangle \end{aligned}$$

Therefore, we calculated twice less inner products in this case than in the case of classical trial functions given by (20).

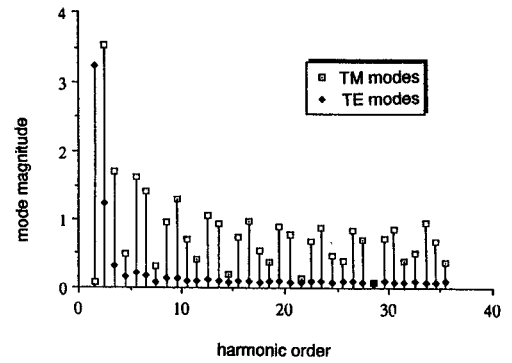
$A_m$  is introduced in (20) to ensure a zero-component of the electric field on the fin (boundary condition).

Nevertheless, for information, we give the variation of the first trial functions along the  $z$ -axis in the discontinuity plane (Fig. 6).

The obtained results with the new kind of trial functions are very encouraging: the spurious solutions  $\beta_\infty$ , detected in



(a)



(b)

Fig. 5. Mode Magnitudes spectrum. (a) Of spurious solution:  $\beta = 61.240, 01$  rad/m. (b) Of physical solution:  $\beta = 598, 82$  rad/m.

the calculation of the propagation constant of the fundamental mode in a unilateral finline, has disappeared, or at least, is removed far away from the investigation domain of physical solutions for  $\beta$  (its value is greater than  $10^8$  rad/m!).

The cosine trial functions, namely:

$$\begin{aligned} \phi_x^m(x) &= \cos(m-1)\frac{2\pi}{w}\left(x - \frac{a}{2}\right) \\ \forall m &\in \{1, 2, 3, \dots, N_x\} \\ \phi_z^k(x) &= \sin(k-1)\frac{2\pi}{w}\left(x - \frac{a}{2}\right) \\ \forall k &\in \{1, 2, 3, \dots, N_z\} \end{aligned} \quad (21)$$

seem not to generate spurious solutions—moreover note that they satisfy the criterion (11)—and do not involve complex calculations. The drawback in the manipulation of this kind of trial functions is that the solutions of the resonance condition (zero determinant) do not converge very well with the number of these trial functions. Since they do not take the edge effects into account, they involve matrices of relatively large sizes.

The variation of the determinant versus the propagation constant  $\beta$  for the three kinds of trial functions (19), (20), and (21) (Fig. 7) shows a very similar behavior of the determinant in the cases of the trial functions (20) and the cosine trial functions.

Finally, calculate the power density of the true and spurious solution in the cross section of a unilateral finline (Fig. 8) (the spurious solution is obtained in the case of trial functions which do not satisfy the criterion (11), that is those of equation (19)).

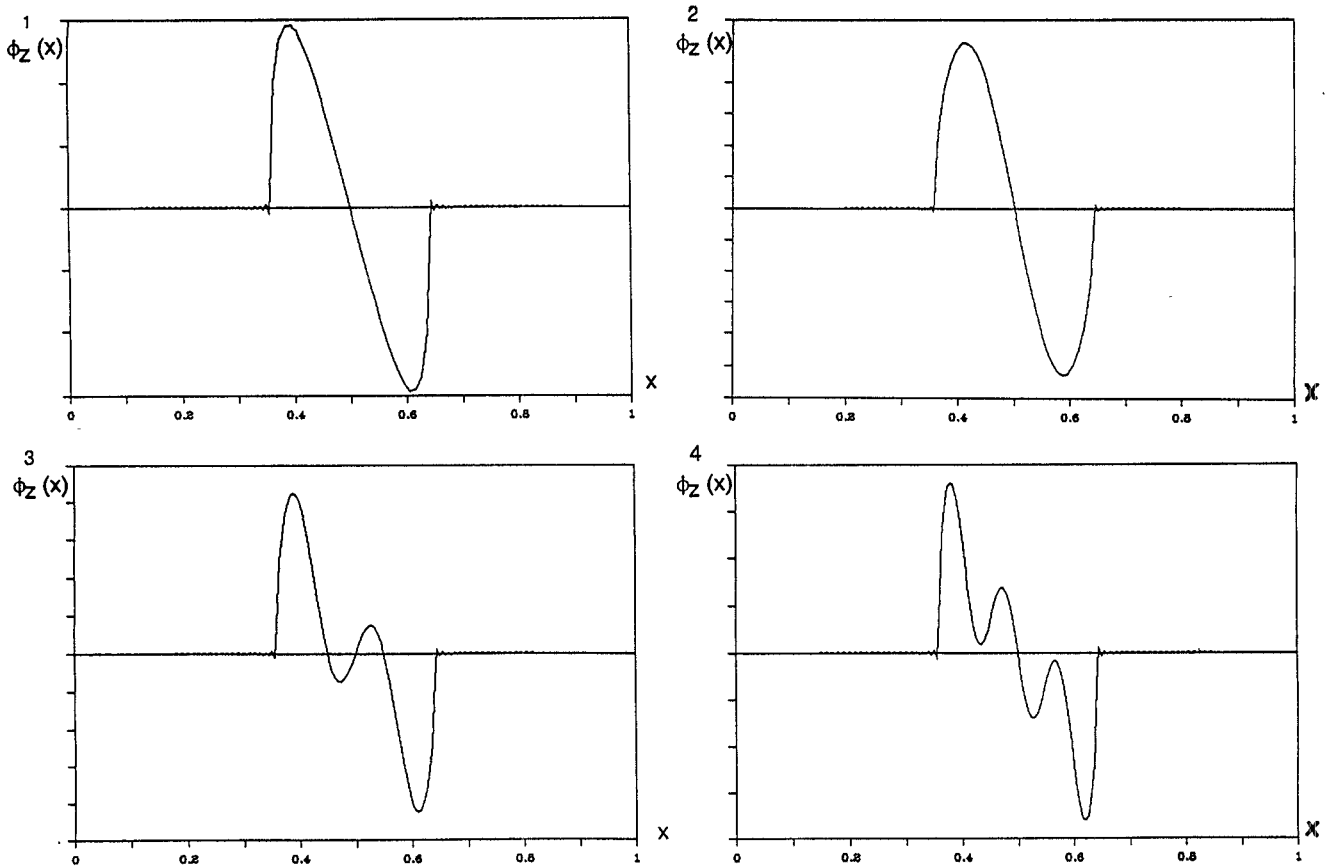


Fig. 6. The new trial functions along the  $z$ -axis in the case of unilateral finline (see Fig. 4).

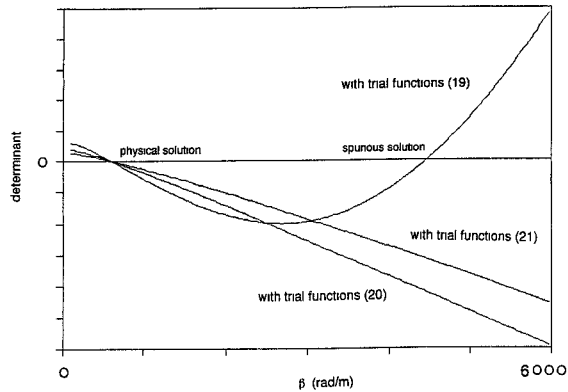


Fig. 7. Determinant versus the unknown  $\beta$  for three kinds of trial functions.

We find that the energy of the physical solution is principally localized between the fins. In contrast, the spurious solution localizes its energy near the fins and essentially, near the edges.

#### IV. CONCLUSION

A very promising and simple criterion about the choice of trial functions in Galerkin's method has been theoretically found to suppress spurious modes present in the transverse resonance method. With a particular choice of trial functions, this method does not suffer from the appearance of nonphysical solutions in the numerical resolution of dispersion problem.

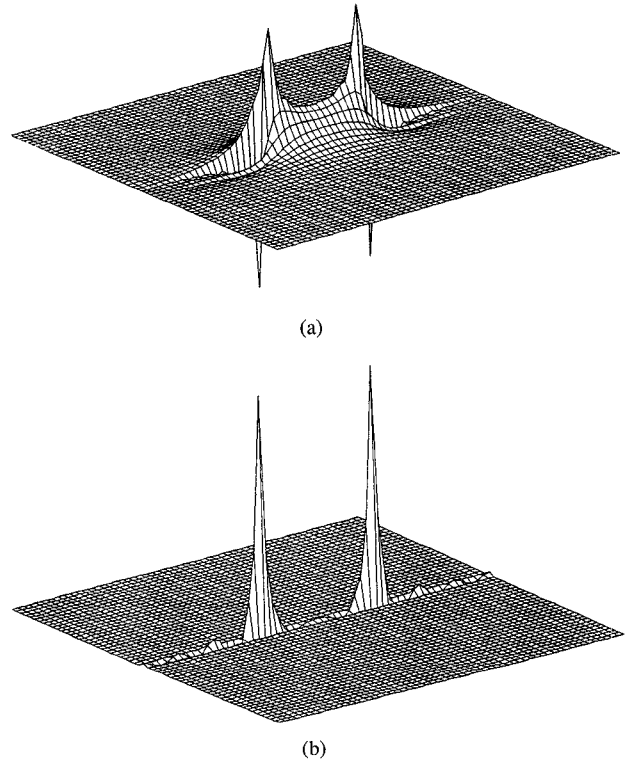


Fig. 8. Power density in the cross section of a unilateral finline. (a) For the physical solution ( $\beta = 598.82$  rad/m). (b) For the spurious solution ( $\beta = 61.240,01$  rad/m).

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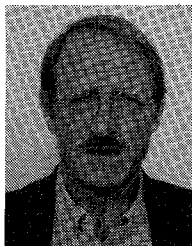
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